SOUND EMISSION IN A NONEQUILIBRIUM MEDIUM FROM A PLANE SURFACE WITH PERIODICALLY VARYING TEMPERATURE

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Any change in temperature T of a continuous medium is accompanied by a change in its density ρ, which gives rise to motion and, in particular, to sound emission. This question, which has been examined for the usual equilibrium medium, in [1], for instance, is extended in the present paper to the case of a nonequilibrium medium.

By a nonequilibrium medium we mean a medium in which there occur processes of energy redistribution among the different degrees of freedom of the particles (translational, internal, chemical), leading to relaxation of the macroscopic characteristics (specific heats c_p and c_v, thermal conductivity $\varkappa$, sound velocity c). The most rapid process is the redistribution of energy among the translational degrees of freedom. The relaxation time of this process $\tau_0 \sim \lambda/c$, where λ is the mean free path. The other energy redistribution processes are much slower. The effects occurring in a nonequilibrium medium can be phenomenologically described within the framework of thermodynamically irreversible processes, where the hydrodynamic equations are supplemented with expressions for the stress tensor of the thermal (q) and diffusion (1) fluxes, and also of the scalar (chemical) flux through the strain velocity tensor, the gradient T, and chemical potential μ. These expressions are obtained by an appropriate choice of fluxes and force from the condition of increase in entropy s, using the Curie principle and Onsager relations [2]. The system of equations is closed by the kinetic equation for the nonequilibrium parameter ξ; the derivative of the characteristic thermodynamic potential with respect to φ is $\varphi\xi = \mu$.

In the linear simultaneous approximation with zero mean velocity v the equation for production of s has the form [2]

$$\rho(Ts_t+\mu\xi_t)+q_x=0, \tag{1}$$

where the subscripts denote partial differentiation. This equation agrees with the more general equation [2] to the accuracy of the small nonlinear term proportional to μ^2. Taking as φ the thermodynamic potential proper in variables p, T, and ξ (p is the pressure), we obtain, for p = const,

$$Ts_t+\mu\xi_t=Ts_{T\infty}T_t+(Ts_\xi+\mu)\xi_t=c_{p\infty}T_t+w_\xi\,\xi_t, \tag{2}$$

where w is the enthalpy, and the subscript ∞ refers to the frozen state (the subscript 0 below refers to the equilibrium state).

For q and I, ignoring the small thermal-diffusion correction, we have the expressions [1, 2]

$$q=-\varkappa_\infty T_x+w_\xi I;\ I=-\rho D\xi_x, \tag{3}$$

where D > 0 is the coefficient of diffusion.

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Finally, the kinetic equation for ξ has the form [2]

$$\rho\xi_t + I_x + M\mu = 0, \tag{4}$$

where $M > 0$ is the kinetic coefficient. For small deviations from equilibrium we perform the expansion

$$\mu = \varphi_\xi = \varphi_{\xi T} T' + \varphi_{\xi\xi}\xi', \tag{5}$$

where the dashes refer to perturbations.

The system of equations (1)-(5) is closed and completely describes the simultaneous processes of heat conduction and diffusion in a nonequilibrium medium.

We establish relations between equilibrium and frozen quantities. For the equilibrium variation of ξ_0' with T we have from (5)

$$\left(\frac{\partial \xi_0}{\partial T}\right)_{p,\mu} = -\frac{\varphi_{\xi T}}{\varphi_{\xi\xi}} = \frac{s_\xi}{\mu_\xi}.$$

Then, from (2) and (3) we obtain

$$\begin{aligned} c_{p0} - c_{p\infty} &= Ts_\xi^2/\mu_\xi > 0, \\ \varkappa_0 - \varkappa_\infty &= \rho D T s_\xi^2/\mu_\xi > 0, \end{aligned} \tag{6}$$

where $\mu\xi > 0$, in view of thermodynamic stability [2]. It follows from (6) that

$$\frac{\varkappa_0 - \varkappa_\infty}{c_{p0} - c_{p0\infty}} = \rho D. \tag{7}$$

Introducing the critirion $L = D/\chi$ [$\chi = \varkappa/(\rho c_p)$ is the thermal diffusivity], we have from (6) and (7) for $L_0 < 1$ the inequalities $L_\infty < L_0 < 1$ or $D < \chi_0 < \chi_\infty$, for $L_0 > 1$ the inequalities $1 < L_0 < L_\infty$ or $\chi_\infty < \chi_0 < D$, i.e., L_0 is always closer to 1 than L_∞. From the estimate $L = (6/5)(\gamma/f)$, where γ is the ratio of specific heats, f is the kinetic function, equal to 5/2 for Maxwellian molecules and having a minimum value of 1.41 for NH_3 [3], it follows that the situation $L_0 < 1$, i.e., $\chi_0 < \chi_\infty$, is more common than the situation $L_0 < 1$.

Putting (4) in the form

$$\tau_1\left(\frac{\partial}{ct} - D\frac{\partial^2}{\partial x^2}\right)\xi + (\xi' - \xi_0') = 0,$$

where

$$\tau_1 = \rho/(M\mu_\xi) > 0 \text{ and}$$

$$\xi_0' = \left(\frac{\partial \xi_0}{\partial T}\right)_{p,\mu} T' = \frac{s_\xi}{\mu_\xi} T',$$

we obtain for g and T the equation

$$\tau_1\left(\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2}\right)(q + \varkappa_\infty T_x) + (q + \varkappa_0 T_x) = 0.$$

For any of the functions $u = T$, ξ, q, and I we have the generalized thermal conductivity equation [$\tau_2 = (c_{p\infty}/c_{p0})\tau_1$]

$$\hat{L}(u) \equiv \left[\tau_2\left(\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial}{\partial t} - \chi_\infty\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial}{\partial t} - \chi_0\frac{\partial^2}{\partial x^2}\right)\right]u = 0. \tag{8}$$

In the non-one-dimensional case $(\partial/\partial x)^2$ can be replaced by the Laplacian; in the presence of transfer $\partial/\partial t$ becomes $\partial/\partial t + (\mathbf{v}\nabla)$. Equation (8) is of the fourth order in coordinate and, hence, we require four boundary conditions, which can be assigned boundary values of T, ξ, and q, I.

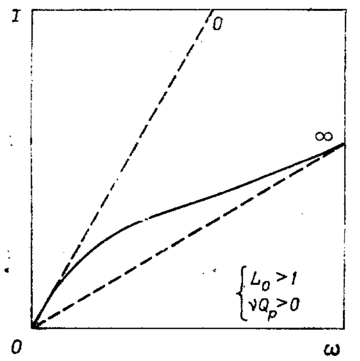

Fig. 1

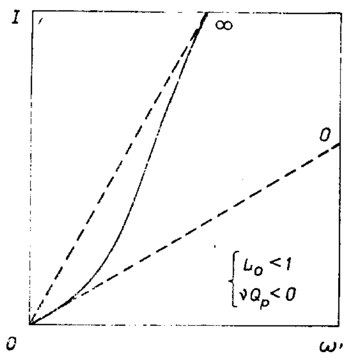

Fig. 2

We turn to the specific problem. Let the small variable part of the temperature of the plane surface vary according to the law $T'_\omega e^{-i\omega t}$, where ω is the oscillation frequency. Then temperature-diffusion waves proportional to $\exp[i(kx - \omega t)]$ will propagate in the nonequilibrium medium. The dispersion equation for the wave number k is obtained from (8) and has the form

$$\tau_2(-i\omega+Dk^2)\,(-i\omega+\chi_\infty k^2)+(-i\omega+\chi_0 k^2)=0.$$

For the decaying waves when $x \to \infty$ we have two solutions with $\mathrm{Im}(k) > 0$. For perturbation of any value of u we have

$$u' = \sum_{l=1}^{2} u'_l e^{i(k_l x - \omega t)}.$$

The perturbation amplitudes u'_l are found from the boundary conditions, for which we take

$$T'(0, t) = T'_w e^{-i\omega t}, \quad I'(0, t) = 0,$$

i.e., we assume that the boundary surface is impermeable to the surrounding substance. The expressions T'_l and ξ'_l are connected by a relation derived from (1)-(3),

$$c_{p\infty}\left(-i\omega + \chi_\infty k_l^2\right) T'_l + w_\xi\left(-i\omega + Dk_l^2\right)\xi'_l = 0.$$

In the special cases where $\tau_2 \to 0$ and ∞, the generated waves are purely temperature waves. In the general case two temperature-diffusion waves propagate.

For the validity of the adopted macroscopic description we must have the inequality $\omega\tau_0 \ll 1$, i.e., in view of the known estimate $\chi \sim c^2\tau_0$, we must have $\omega \ll c^2/\chi$, which means a low-frequency approximation. On the other hand, this means that the perturbation wavelength $1/k \sim \sqrt{\chi/\omega} \ll c/\omega$ – the length of the sound wave. Thus, we arrive at the separation of the considered temperature-diffusion and hydrodynamic (acoustic) problems [1]. In particular, the boundary value of v' in the hydrodynamic problem will be the value of v' when $x \to \infty$ in the temperature-diffusion problem.

The relationship connecting U' with T' and ξ' is obtained from the continuity equation

$$\rho v'_x = -\rho'_t = -\rho_{T\infty} T'_t - \rho_\xi \xi'_t = i\omega\left(\rho_{T\infty} T' + \rho_\xi \xi'\right),$$

so that when $v'(0, t) = 0$ we have

$$v'(\infty, t) = i\omega\left(\rho_{T\infty}\int_0^\infty T'dx + \rho_\xi\int_0^\infty \xi'dx\right)\Big/\rho.$$

The required intensity of sound emission from unit surface is [1]

$$I=\rho c\langle|v'(\infty,t)|^2\rangle.$$

In the general case the solution has the form ($\Omega = -i\omega$)

$$v'(\infty, t) = -\Omega\beta_\infty T'_w e^{\Omega t}\frac{k_1 + k_2}{ik_1k_2}\frac{1 + (\chi_\infty - \chi_0)\,\delta/(D\varepsilon)}{1 + (\chi_\infty - D)\,\tau_2k_1k_2/\varepsilon}, \tag{9}$$

where

$$\beta_\infty = -\rho_{T\infty}/\rho;\quad \sigma = (c_{p\infty}/\rho_{T\infty})(\rho_\xi/w_\xi);$$
$$\varepsilon = (1/L_0 - 1) + (1/L_\infty - 1)\,\Omega\tau_2;$$
$$k_1k_2 = \sqrt{\frac{\Omega(1 + \Omega\tau_2)}{\tau_2 D\chi_\infty}}.$$

When $\omega \to 0$ and ∞(9) takes the form [1]

$$v'(\infty, t) \to -\zeta\sqrt{\Omega}\,T'_w e^{\Omega t},$$

where $\zeta = \beta\sqrt{\chi}$ has equilibrium and frozen values, respectively. For proof of the limiting expressions we use the relations

$$\frac{\chi_\infty - \chi_0}{c_{p0} - c_{p\infty}} = \frac{\chi_\infty - D}{c_{p0}} = \frac{\chi_0 - D}{c_{p\infty}},$$

which follow from [7], and the thermodynamic relation

$$\frac{\rho_{T0} - \rho_{T\infty}}{c_{p0} - c_{p\infty}} = \frac{\rho_\xi}{w_\xi}.$$

Interpreting ξ as the extent of the chemical reaction, we obtain for an ideal gas

$$\frac{\beta_0 - \beta_\infty}{\beta_\infty} = \nu\Phi\frac{Q_p}{RT}, \tag{10}$$

where R is the gas constant: $\nu = \sum_k \nu_k$ is the overall stoichiometric coefficient of the reaction; $Q_p = \sum_k \nu_k W_k$ is its molar thermal effect when p = const,

$$\frac{1}{\Phi} = \sum_k \frac{\nu_k^2}{x_k} - \nu^2 = \frac{1}{2}\sum_{k,k'}\frac{(\nu_k x_{k'} - \nu_{k'}x_k)^2}{x_k x_{k'}} > 0,$$

x_k is the mole fraction of the component K_k.

The maximum value of $|\nu|\Phi$ is

$$|\nu|\Phi_{\max} = \frac{1}{4}\left|\frac{1}{\sum\limits_{k_-}\nu_{k_-}} + \frac{1}{\sum\limits_{k_+}\nu_{k_+}}\right|,$$

where $\nu_{k_+} = \lessgtr 0$. For instance, for the dissociation reaction $K_1 + K_2 + K_3 = 0$, Expression (10) has the form

$$\left|\frac{\beta_0 - \beta_\infty}{\beta_\infty}\right| = \frac{\alpha(1 - \alpha)}{2}\left|\frac{Q_p}{RT}\right| \leqslant \frac{1}{8}\left|\frac{Q_p}{RT}\right|,$$

where α is the degree of dissociation; $Q_p = -W_1 + W_2 + W_3$.

It follows from the presented expressions that when $L_0 \lessgtr 1$ and $\nu Q_p \lessgtr 0$ we have $\zeta_0 \lessgtr \zeta_\infty$. For reactions involving a fairly large heat effect at not very high temperatures the difference between ζ_0 and ζ_∞ can be fairly appreciable, so that the temperature generation of low-frequency sound (infrasound) in an equilibrium-reacting medium may differ significantly in principle from that in an ordinary medium. This is illustrated by Figs. 1 and 2, where sample relationships between the intensity of sound and its frequency, based on two different assumptions, are shown.

LITERATURE CITED

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